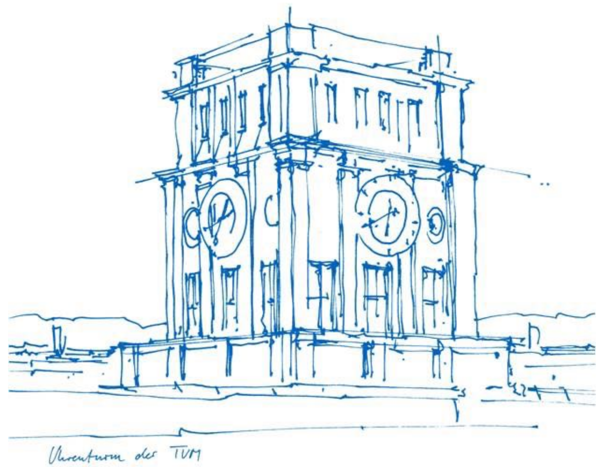
Experimental Substructuring for Linear and Nonlinear Connection Dynamics: A Tutorial

Part 1: linear connection



Daniel Rixen - Technical University of Munich

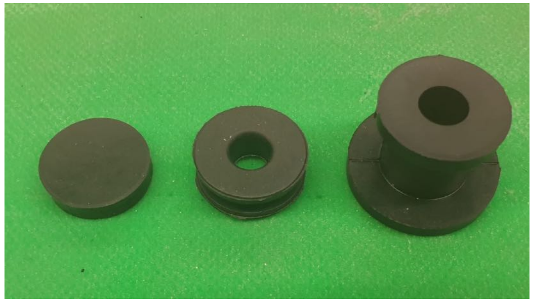
H. Nevzat Özgüven – Middle East Technical University (Part 2 this afternoon 2pm)

“ The dynamics of joints is one of the main remaining challenges in structural dynamics ”

or something along that line …

Dave Ewins

[2]



This presentation assumes the joint dynamics to be linear.
For non-linear joint identification by substructuring, see Session 08 (N. Özgüven), 2:00 pm, Salon 8

[2] Brake, Matthew RW, and Pascal Reuß. "The Brake-Reuß beams: a system designed for the measurements and modeling of variability and repeatability of jointed structures with frictional interfaces." *The Mechanics of Jointed Structures: Recent Research and Open Challenges for Developing Predictive Models for Structural Dynamics* (2018): 99-107.

Since joints is about assemblies of components, substructuring could help analyze joints …

Based on work done over the last 10 years with

Dennis de Klerk

Sven Voormeeren

Marteen vdSeijs

Michael Häußler

Steven Klaassen

Ahmed El Mahmoudi

Michael Kreutz

Francesco Trainotti

Verena Gimpl

Mert Göldeli

Overview of Tutorial

- Basics of Substructuring
  - Primal and dual assembly
  - Frequency-Based Substructuring
  - Some tricks - Measure more then you need
  - Some tricks - Decoupling
  - Some tricks - Including interface dynamics (transmission simulator)

- Interface Identification
  - Method 1: Dual Decoupling
  - Method 2: LM-FBS with weak interface
  - Method 3: Inverse Substructuring
  - Summary of methods

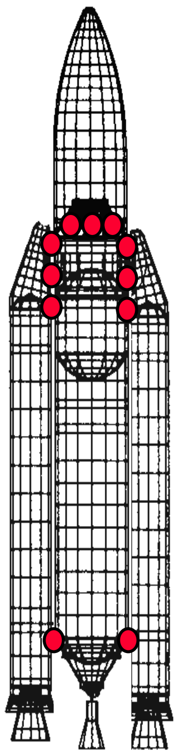
- Example: Rubber joint
- Example: Bolted joint

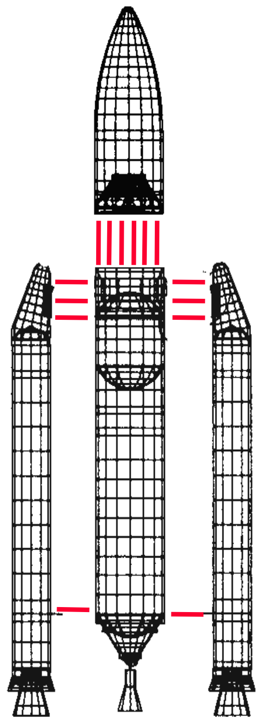
Basics of substructuring

Primal assembly

$$\begin{bmatrix} \mathbf{K}^{(1)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{K}^{(N)} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(N)} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \vdots \\ \mathbf{f}^{(N)} \end{bmatrix}$$

$$\boxed{\mathbf{Z}_g \mathbf{u}_g = \mathbf{f}_g}$$





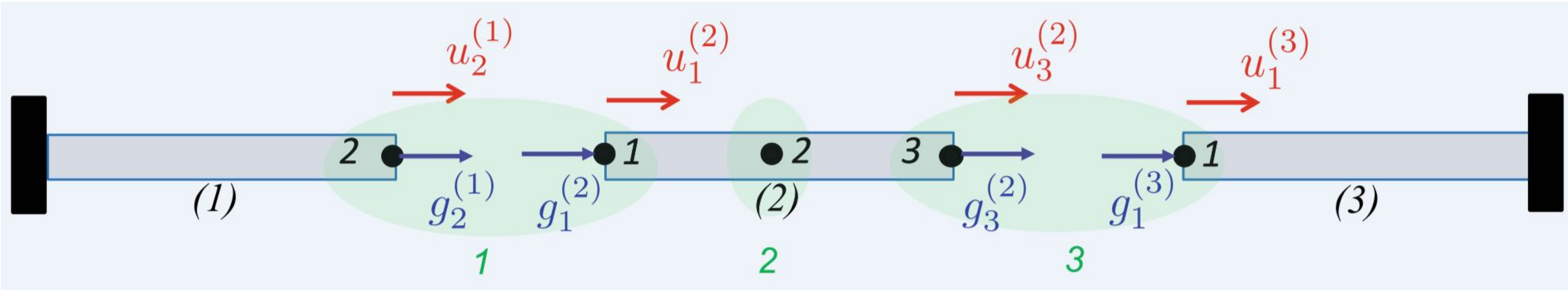
Dual assembly

$$\begin{bmatrix} \mathbf{K}^{(1)} & & \mathbf{0} & \mathbf{B}^{(1)^T} \\ & \ddots & & \\ \mathbf{0} & & \mathbf{K}^{(N)} & \mathbf{B}^{(N)^T} \\ \mathbf{B}^{(1)} & \cdots & \mathbf{B}^{(N)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(N)} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \vdots \\ \mathbf{f}^{(N)} \\ \mathbf{0} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \mathbf{Z} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{u}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{f}} \\ \mathbf{0} \end{bmatrix}}$$

Basics of substructuring

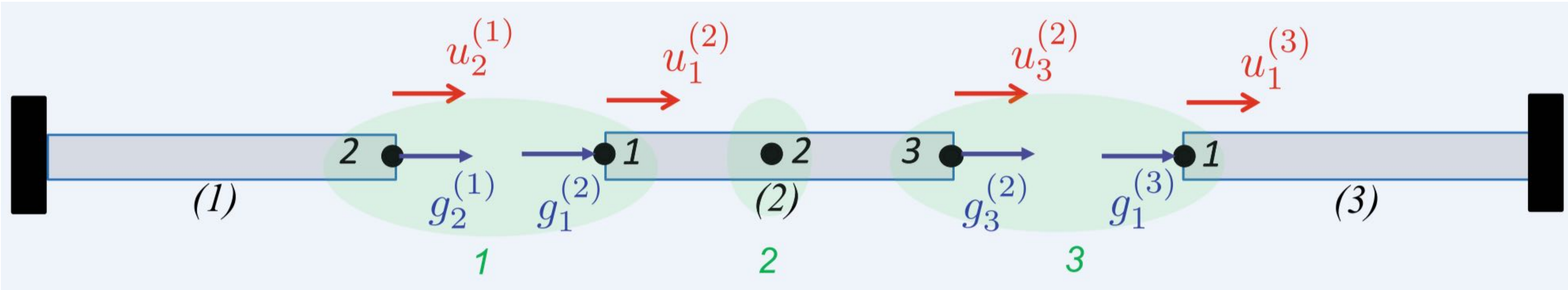
Example of primal assembly



$$\begin{bmatrix} Z_{22}^{(1)}+Z_{11}^{(2)} & Z_{12}^{(2)} & Z_{13}^{(2)} \\ Z_{21}^{(2)} & Z_{22}^{(2)} & Z_{23}^{(2)} \\ Z_{31}^{(2)} & Z_{32}^{(2)} & Z_{11}^{(2)}+Z_{33}^{(3)} \end{bmatrix} \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{bmatrix} = \begin{bmatrix} \bar{f}_2^{(1)}+\bar{f}_1^{(2)} \\ \bar{f}_2^{(2)} \\ \bar{f}_3^{(2)}+\bar{f}_1^{(3)} \end{bmatrix}$$

Basics of substructuring

Example of dual assembly



$$
\begin{bmatrix}
[Z_{22}^{(1)}] & \mathbf{0} & \mathbf{0} & [1 \;\; 0] \\
\mathbf{0} & \begin{bmatrix} Z_{11}^{(2)} & Z_{12}^{(2)} & Z_{13}^{(2)} \\ Z_{21}^{(2)} & Z_{22}^{(2)} & Z_{23}^{(2)} \\ Z_{31}^{(2)} & Z_{32}^{(2)} & Z_{33}^{(2)} \end{bmatrix} & \mathbf{0} & \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \\
\mathbf{0} & \mathbf{0} & [Z_{11}^{(3)}] & [0 \;\; -1] \\
\begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \end{bmatrix} & \mathbf{0}
\end{bmatrix}
\begin{bmatrix}
[\bar{u}_2^{(1)}] \\
\begin{bmatrix} \bar{u}_1^{(2)} \\ \bar{u}_2^{(2)} \\ \bar{u}_3^{(2)} \end{bmatrix} \\
[\bar{u}_1^{(3)}] \\
\lambda_1 \\
\lambda_2
\end{bmatrix}
=
\begin{bmatrix}
[\bar{f}_2^{(1)}] \\
\begin{bmatrix} \bar{f}_1^{(2)} \\ \bar{f}_2^{(2)} \\ \bar{f}_3^{(2)} \end{bmatrix} \\
[\bar{f}_1^{(3)}] \\
0 \\
0
\end{bmatrix}
$$

Basics of substructuring

Dual assembly using admittances

Measured
in experimental tests

$$\begin{bmatrix} \mathbf{Z} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \longrightarrow \mathbf{u} = \mathbf{Y}(\mathbf{f} - \mathbf{B}^T \boldsymbol{\lambda})$$

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}^{(1)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{Z}^{(N^{sub})} \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{(1)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{Y}^{(N^{sub})} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}^{(1)^{-1}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{Z}^{(N^{sub})^{-1}} \end{bmatrix} = \mathbf{Z}^{-1}$$

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(N^{sub})} \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \vdots \\ \mathbf{f}^{(N^{sub})} \end{bmatrix} \qquad \boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_{n_\lambda} \end{bmatrix}$$

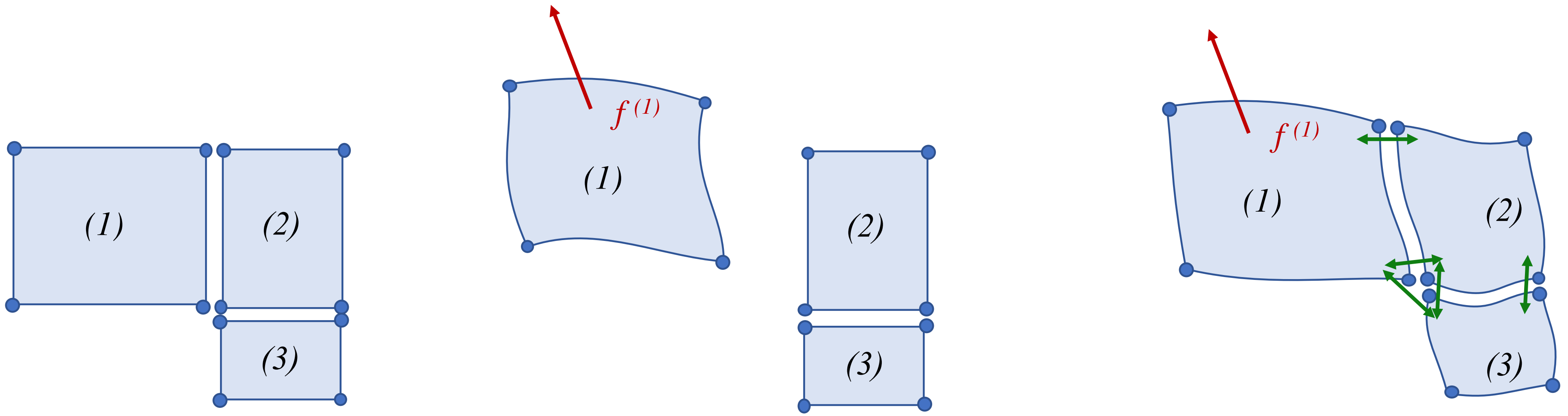
Basics of substructuring

Dual assembly using admittances

$$\begin{bmatrix} \mathbf{Z} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \longrightarrow \mathbf{u} = \mathbf{Y}(\mathbf{f} - \mathbf{B}^T \boldsymbol{\lambda})$$

$$\longrightarrow (\mathbf{B}\mathbf{Y}\mathbf{B}^T)\boldsymbol{\lambda} = \mathbf{B}\mathbf{Y}\mathbf{f}$$

"dual interface problem"



a. Substructured problem

b. Effect of local force in $\Omega^{(1)}$

c. Global effect of $\boldsymbol{\lambda}$ to enforce compatiblity

Basics of substructuring

Dual assembly using admittances

$$(\mathbf{BYB}^T)\boldsymbol{\lambda} = \mathbf{BYf}$$

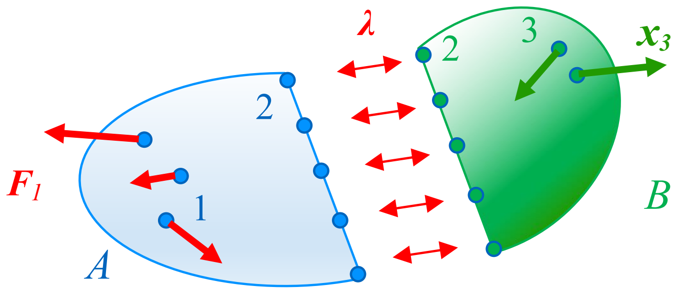
“dual interface problem”

$$\boldsymbol{\lambda} = (\mathbf{BYB}^T)^{-1}\mathbf{BYf}$$

$$\mathbf{u} = \mathbf{Y}(\mathbf{f} - \mathbf{B}^T\boldsymbol{\lambda})$$

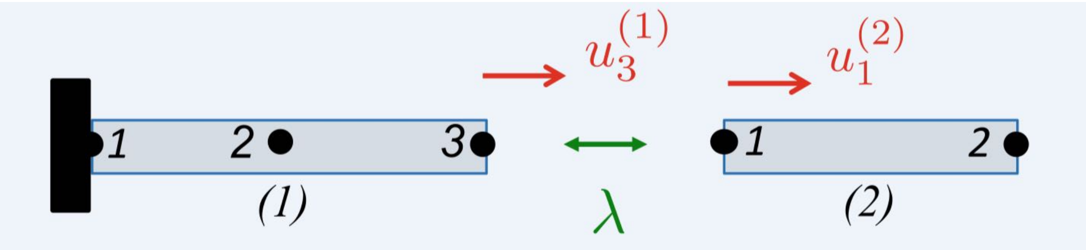
$$\boxed{\mathbf{u} = \underbrace{\left(\mathbf{Y} - \mathbf{YB}^T(\mathbf{BYB}^T)^{-1}\mathbf{BY}\right)}_{\mathbf{Y}_{g,dual}}\mathbf{f}}$$

Lagrange Multiplier FBS (LM-FBS)



Interpretation: $\mathbf{u} = \Big(\overbrace{\mathbf{Y}}^{uncoupl.}\ \overbrace{-\mathbf{YB}^T\underbrace{(\mathbf{BYB}^T)^{-1}\underbrace{\mathbf{BY}}_{\Delta}}_{\lambda}}^{coupling}\Big)\mathbf{f}$

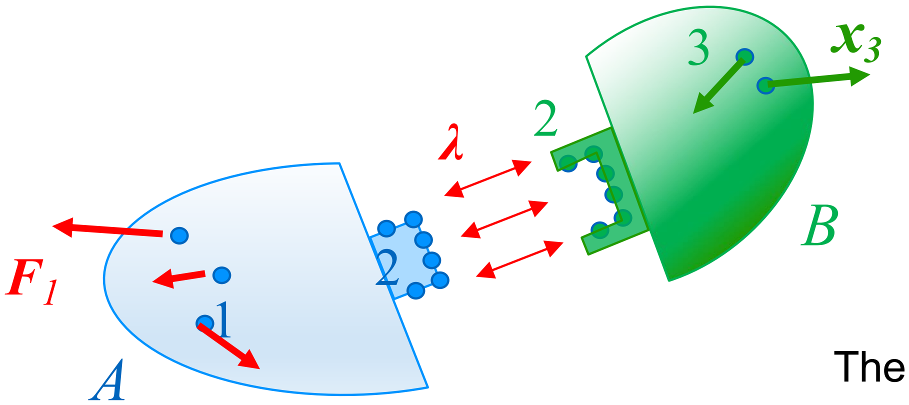
Basics of substructuring

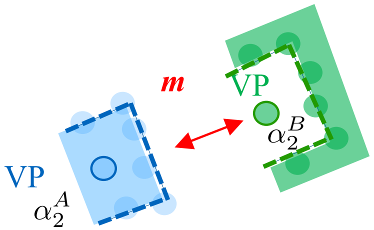


$$\mathbf{Y}_{assemb.} = \begin{bmatrix} Y_{22}^{(1)} & Y_{23}^{(1)} & 0 & 0 \\ Y_{32}^{(1)} & Y_{33}^{(1)} & 0 & 0 \\ 0 & 0 & Y_{11}^{(2)} & Y_{12}^{(2)} \\ 0 & 0 & Y_{21}^{(2)} & Y_{22}^{(2)} \end{bmatrix} - \begin{bmatrix} Y_{23}^{(1)} \\ Y_{33}^{(1)} \\ -Y_{11}^{(2)} \\ -Y_{21}^{(2)} \end{bmatrix} \left(Y_{33}^{(1)} + Y_{11}^{(2)}\right)^{-1} \begin{bmatrix} Y_{32}^{(1)} & Y_{33}^{(1)} & -Y_{11}^{(2)} & -Y_{12}^{(2)} \end{bmatrix}.$$

Basics of substructuring

Tricks in dual assembly – Measure more then you need





$$\boldsymbol{x}_2^A = \boldsymbol{R}_2^A \alpha_2^A + \mu^A$$

$$\rightarrow\ \alpha_2^A = \left(\boldsymbol{R}_2^{A^T} \boldsymbol{R}_2^A\right)^{-1} \boldsymbol{R}_2^{A^T} \boldsymbol{x}_2^A = \boldsymbol{T}_{VPT}^A \, \boldsymbol{x}_2^A$$

Transformation matrix that computes in a least-square sense the rigid motion amplitudes

The same is done for the other side of the interface

→ compatibility in enforced as $\alpha_2^A = \alpha_2^B$

i.e. all interface displacements shapes that are not rigid are left unassembled

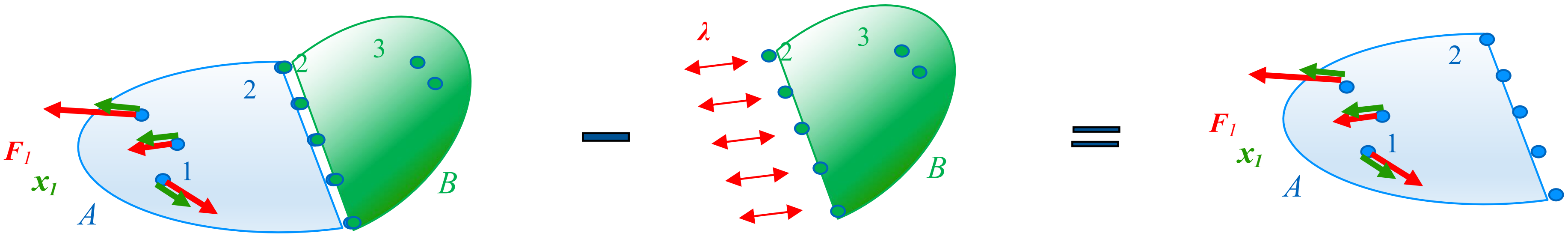
→ Much better conditioning of the interface problem

"Virtual Point Transformation" (VPT)

M. van der Seijs. Experimental Dynamic Substructuring, Analysis and Design Strategies for Vehicle Development. PhD thesis, Delft University of Technology, Delft, The Netherlands, June 2016.

Basics of substructuring

Tricks in dual assembly – decoupling



$$\mathbf{u} = \left(\mathbf{Y} - \mathbf{Y}\mathbf{B}^T(\mathbf{B}\mathbf{Y}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{Y}\right)\mathbf{f}$$

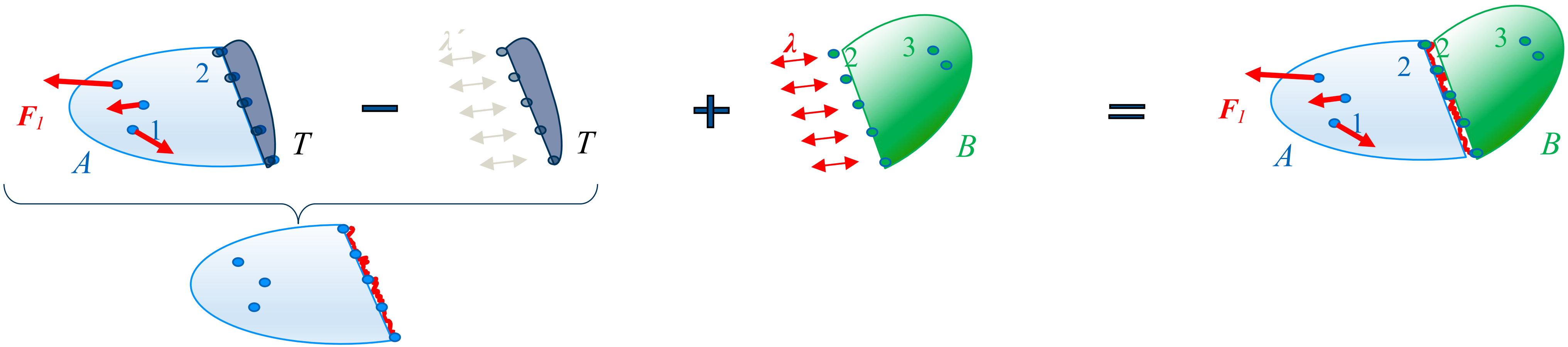
Same formula as before but with a negative substructure

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{(1)} & & \mathbf{0} \\ & -\mathbf{Y}^{(B)} & \\ & \ddots & \\ \mathbf{0} & & \mathbf{Y}^{(N^{sub})} \end{bmatrix}$$

Basics of substructuring

Tricks in dual assembly – including interface dynamics (transmission simulator)

To include the interface dynamics in the coupling, one identifies one of the substructure with the interface dynamics activated by a transmission simulator:



Cool! …

…. So can we also isolate the interface dynamics alone ??

* The name “transmission simulator” was first coined in
R. L. Mayes and M. Arviso. Design studies for the transmission simulator method of experimental dynamic substructuring. In International seminar on modal analysis, ISMA, Leuven, 2010. KUL.

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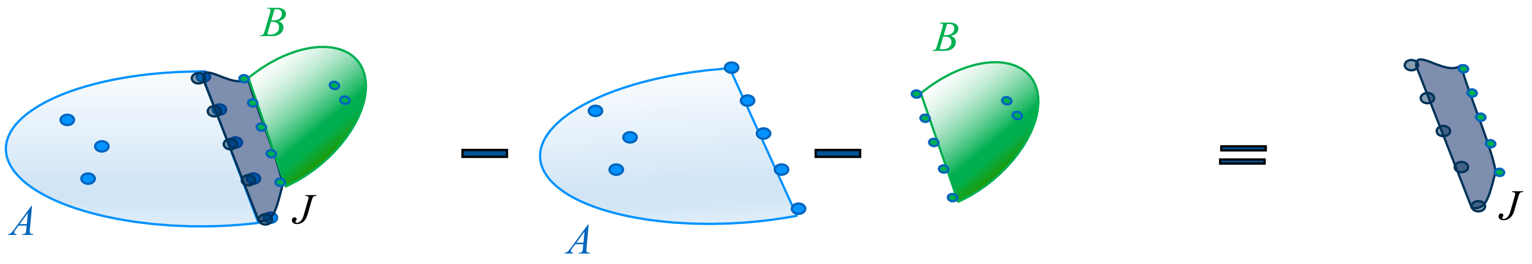
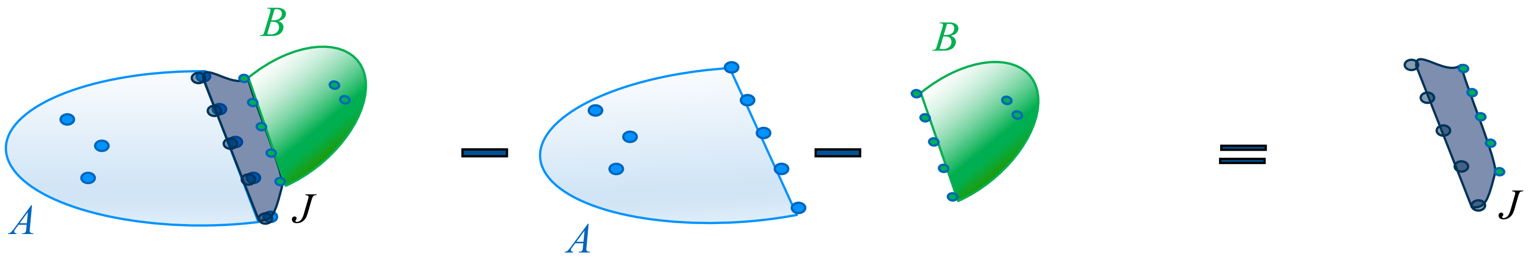
Interface identification: Method 1. : Dual Decoupling



- Measure $\boldsymbol{Y}_{qm}^{AJB}, \boldsymbol{Y}_{qm}^{A}, \boldsymbol{Y}_{qm}^{B}$ (including usually a VPT for interface dofs)
- Assemble negative A and B substructures to AJB (Decoupling)

$$\boldsymbol{Y}_{\text{decoupled}}^{J} = \left(\boldsymbol{I} - \boldsymbol{Y}\boldsymbol{B}^{T}\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^{T}\right)^{-1}\boldsymbol{B}\right)\boldsymbol{Y} \qquad \text{with} \quad \boldsymbol{Y} = \begin{pmatrix} \boldsymbol{Y}_{qm}^{AJB} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{Y}_{qm}^{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Y}_{qm}^{B} \end{pmatrix}$$

Interface identification: Method 1. : Dual Decoupling



\+ General technique, that can also isolate "dynamic" joints

Can be made more robust (e.g. forcing also internal dofs of subtracting substructures to be compatible) [1,2]

\- The decoupling problem is often badly conditioned because J typically much stiffer than A and B

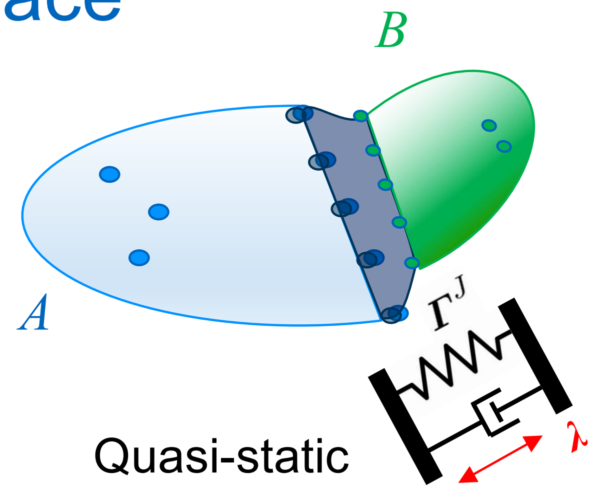
➡ Consider the interface a priori as being very stiff ("*quasi-static*")

Needs to measure the assembly AJB and the individual substructure than A and B .

➡ If quasi-static, the joint behaviour can be find without needing to measure A and B

[1] P. Sjövall et al. Component system identification and state-space model synthesis. *Mechanical Systems and Signal Processing*, 21:2697–2714, 2007.
[2] S. Voormeeren et al. Substructure decoupling techniques - a review and uncertainty propagation analysis. In *IMAC-XXVII: International Modal Analysis Conference, Orlando, FL*, Bethel, CT, February 2009. SEM

Interface identification:
Method 2. : LM-FBS with weak interface



As before: $\mathbf{u} = \mathbf{Y}(\mathbf{f} - \mathbf{B}^T\boldsymbol{\lambda})$

but now no dofs of J involved

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}^A & \\ & \boldsymbol{Y}^B \end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}^A \\ \boldsymbol{u}^B \end{bmatrix}$$

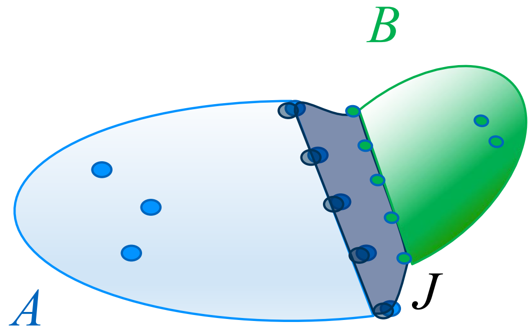
and the compatibility is made "flexible"

$$\boldsymbol{B}\boldsymbol{u} = \boldsymbol{\Gamma}^J \boldsymbol{\lambda}$$

$$\boxed{\boldsymbol{Y}^{AJB} = \boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{B}^T \left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T + \boldsymbol{\Gamma}^J\right)^{-1} \boldsymbol{B}\boldsymbol{Y}}$$

Interface flexibility
(dimension: # of interface compatibility conditions)
Not the same thing as $\boldsymbol{Y}^J$ but related

$$\boldsymbol{Y}^{AJB} = \boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{B}^T\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T + \boldsymbol{\Gamma}^J\right)^{-1}\boldsymbol{B}\boldsymbol{Y}$$



Can be rearranged to compute $\boldsymbol{\Gamma}^J$ without the need to know $\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}^A & \\ & \boldsymbol{Y}^B \end{bmatrix}$!
(e.g. [1,2,3] and tutorial of N. Özguven)

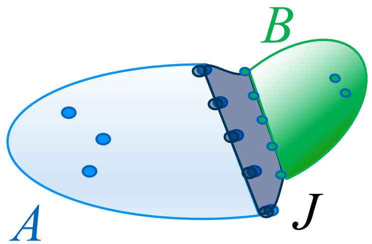
Just to make you believe that this can work, see next slides ...

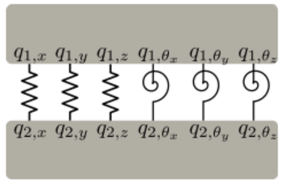
Historically, these methods were called *"inverse substructuring"* (here in its dual form) because the coupling equations are "inverted" to extract $\boldsymbol{\Gamma}^J$

[1]J. Zhen, T. C. Lim, and G. Lu. Determination of system vibratory response characteristics applying a spectral-based inverse sub-structuring approach. part i: analytical formulation. International journal of vehicle noise and vibration, 1(1):1–30, 2004.
[2]Čelič, D., & Boltežar, M. (2008). Identification of the dynamic properties of joints using frequency–response functions. *Journal of Sound and Vibration*, *317*(1-2), 158-174.
[3]TOL, Şerife, et al. Dynamic characterization of bolted joints using FRF decoupling and optimization. Mechanical Systems and Signal Processing, 2015, 54. Jg., S. 124-138.

Interface identification:
Method 3. : Inverse Substructuring (primal)

$$\boxed{\boldsymbol{Y}^{AJB} = \boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{B}^T\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T + \boldsymbol{\Gamma}^J\right)^{-1}\boldsymbol{B}\boldsymbol{Y}}$$





It can be shown that (under same conditions), the equation above can be transformed into

$$\underbrace{\boldsymbol{Z}_{22}^{AJB}}_{\left(\boldsymbol{Y}_{22}^{AJB}\right)^{-1}} = \underbrace{\boldsymbol{Z}_{22}^{AB} + \boldsymbol{Z}_{22}^{J}}_{\begin{bmatrix} \boldsymbol{Z}_{22}^{A} & 0 \\ 0 & \boldsymbol{Z}_{22}^{B} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z}^{J} & -\boldsymbol{Z}^{J} \\ -\boldsymbol{Z}^{J} & \boldsymbol{Z}^{J} \end{bmatrix}} = \begin{bmatrix} \boldsymbol{Z}_{22}^{A} + \boldsymbol{Z}^{J} & -\boldsymbol{Z}^{J} \\ -\boldsymbol{Z}^{J} & \boldsymbol{Z}_{22}^{B} + \boldsymbol{Z}^{J} \end{bmatrix}$$

Measure and invert

$$\boldsymbol{Z}^J = (\boldsymbol{\Gamma}^J)^{-1}$$

Read the off-diagonals

BONUS: Subtract from diagonals to find the dynamics of the substructures!

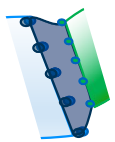
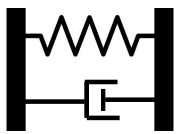
Moorhouse, A., Elliot, A.S., Heo, Y.: Intrinsic characterisation of structure-borne sound sources and isolators from in-situ measurements. In: Proceedings of Meetings on Acoustics ICA2013, vol. 19, p. 065053. ASA (2013)

Meggitt, J.W.R., Elliott, A.S., Moorhouse, A.T., Lai, H.K.: In situ determination of dynamic stiffness for resilient elements. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science p. 0954406215618986 (2015)

Keersmaekers, L., Mertens, L., Penne, R., Guillaume, P., Steenackers, G.: Decoupling of mechanical systems based on in-situ frequency response functions: The link-preserving, decoupling method. Mechanical Systems and Signal Processing (2015)

Interface identification: Summary

Similar or equivalent

joint \ assembly	**dual**	**primal**
dynamic 	Dual Decoupling	Primal Decoupling
quasi-static 	Dual Inv.Substr.	Primal Inv.Substr.

Dual Decoupling

$$\boldsymbol{Y}^J_{\text{decoupled}} = \left(\boldsymbol{I} - \boldsymbol{Y}\boldsymbol{B}^T\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T\right)^{-1}\boldsymbol{B}\right)\boldsymbol{Y}$$

$$\boldsymbol{Y} = \begin{pmatrix} \boldsymbol{Y}^{AJB}_{qm} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{Y}^{A}_{qm} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Y}^{B}_{qm} \end{pmatrix}$$

Primal Decoupling

$$\left(\boldsymbol{Y}_{22}^{AJB}\right)^{-1} = \begin{bmatrix} \boldsymbol{Z}_{22}^A + \boldsymbol{Z}_{AA}^J & -\boldsymbol{Z}_{AB}^J \\ -\boldsymbol{Z}_{BA}^J & \boldsymbol{Z}_{22}^B + \boldsymbol{Z}_{BB}^J \end{bmatrix}$$

Dynamics of A & B needed !

Dual Inv.Substr.

$$\boldsymbol{Y}^{AJB} = \boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{B}^T\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T + \boldsymbol{\Gamma}^J\right)^{-1}\boldsymbol{B}\boldsymbol{Y}$$

Primal Inv.Substr.

$$\left(\boldsymbol{Y}_{22}^{AJB}\right)^{-1} = \begin{bmatrix} \boldsymbol{Z}_{22}^A + \boldsymbol{Z}^J & -\boldsymbol{Z}^J \\ -\boldsymbol{Z}^J & \boldsymbol{Z}_{22}^B + \boldsymbol{Z}^J \end{bmatrix}$$

sometimes called “in-situ”

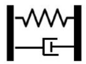
Assumptions on joint !

Interface identification: Remarks

1. Isolation of the joint

Separating the dynamics of the joints from the dynamics of the assembly

→ Substructuring approach to find $\boldsymbol{Y}^J$ from $\boldsymbol{Y}^{AJB}$

joint \ assembly	dual	primal
dynamic 	Dual Decoupling $\boldsymbol{Y}^J_{\text{decoupled}} = \left(\boldsymbol{I} - \boldsymbol{Y}\boldsymbol{B}^T\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T\right)^{-1}\boldsymbol{B}\right)\boldsymbol{Y}$ $\boldsymbol{Y} = \begin{pmatrix} \boldsymbol{Y}^{AJB}_{qm} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{Y}^{A}_{qm} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Y}^{B}_{qm} \end{pmatrix}$	Primal Decoupling $\left(\boldsymbol{Y}^{AJB}_{22}\right)^{-1} = \begin{bmatrix} \boldsymbol{Z}^A_{22} + \boldsymbol{Z}^J_{AA} & -\boldsymbol{Z}^J_{AB} \\ -\boldsymbol{Z}^J_{BA} & \boldsymbol{Z}^B_{22} + \boldsymbol{Z}^J_{BB} \end{bmatrix}$
quasi-static 	Dual Inv.Substr. $\boldsymbol{Y}^{AJB} = \boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{B}^T\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T + \boldsymbol{\Gamma}^J\right)^{-1}\boldsymbol{B}\boldsymbol{Y}$	Primal Inv.Substr. $\left(\boldsymbol{Y}^{AJB}_{22}\right)^{-1} = \begin{bmatrix} \boldsymbol{Z}^A_{22} + \boldsymbol{Z}^J & -\boldsymbol{Z}^J \\ -\boldsymbol{Z}^J & \boldsymbol{Z}^B_{22} + \boldsymbol{Z}^J \end{bmatrix}$

2. Parameterization of joint dynamics

Guess physical parameters from the isolated joint dynamics by fitting a physical model

$$\boldsymbol{Z}^J(\Omega) = \left(-\Omega^2\boldsymbol{M}^J + j\Omega\boldsymbol{C}^J + \left(\boldsymbol{K}^J + j\bar{\boldsymbol{C}}^J\right)\right)$$

Not discussed here:

- Isolation techniques using a model of the substructures (e.g. with SEMM [1])
- If a priori a good knowledge of the joint physics one can also parametrize the joint first and determine them from the measurement by optimization [2]

[1] S. W. B. Klaassen and D. J. Rixen. Using semm to identify the joint dynamics in multiple degrees of freedom without measuring interfaces. In *IMAC-XXXVII: International Modal Analysis Conference, Orlando, FL*, Bethel, CT, January 2019. SEM

[2] Estimating Linear Joint Stiffness and Damping using Frequency-Based Optimization Framework — Marie Brøns, IMAC 2024 (just after this talk ;-)

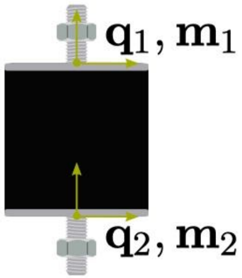
Overview of Tutorial

- Basics of Substructuring
  - Primal and dual assembly
  - Frequency-Based Substructuring
  - Some tricks - Measure more then you need
  - Some tricks - Decoupling
  - Some tricks - Including interface dynamics (transmission simulator)

- Interface Identification
  - Method 1: Dual Decoupling
  - Method 2: LM-FBS with weak interface
  - Method 3: Inverse Substructuring
  - Summary of methods

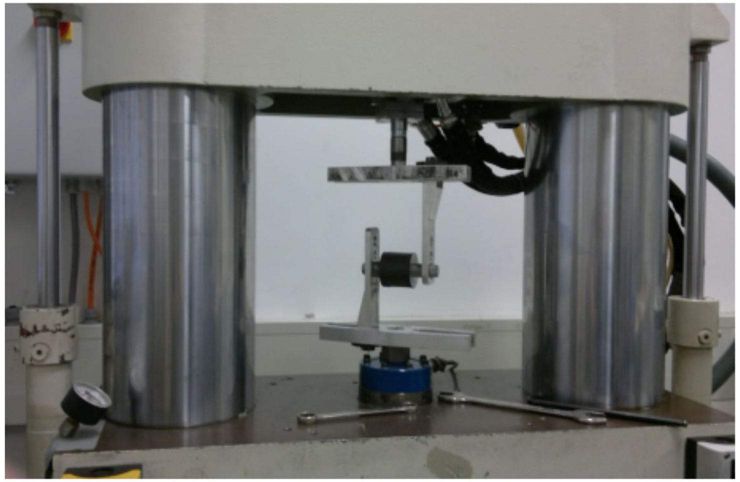
- Example: Rubber joint
- Example: Bolted joint

Example: Rubber joint [1]



Rubber mount used in automotive application

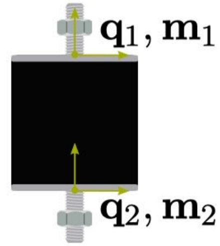
Tested with A & B as steel crosses



Reference from a hydropulse measurement

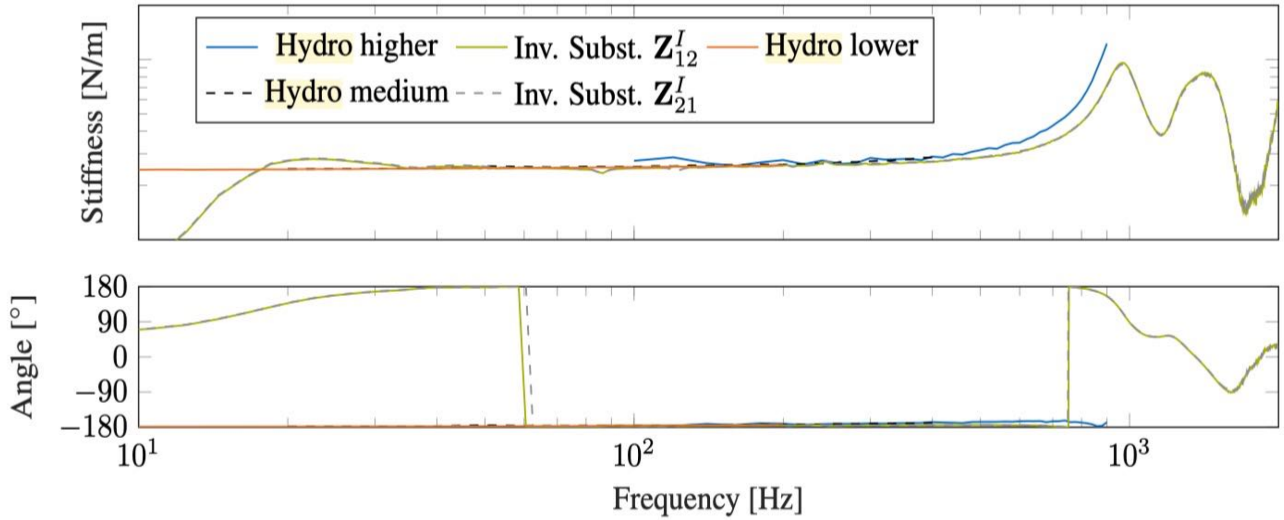
[1] M. Haeussler, S. Klaassen, and D. Rixen. Experimental twelve degree of freedom rubber isolator models for use in substructuring assemblies. *Journal of Sound and Vibration*, 474:115253, 2020.

Example: Rubber joint



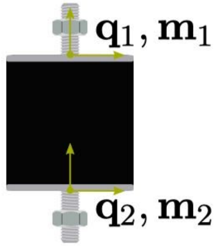
Primal Inv.Substr.

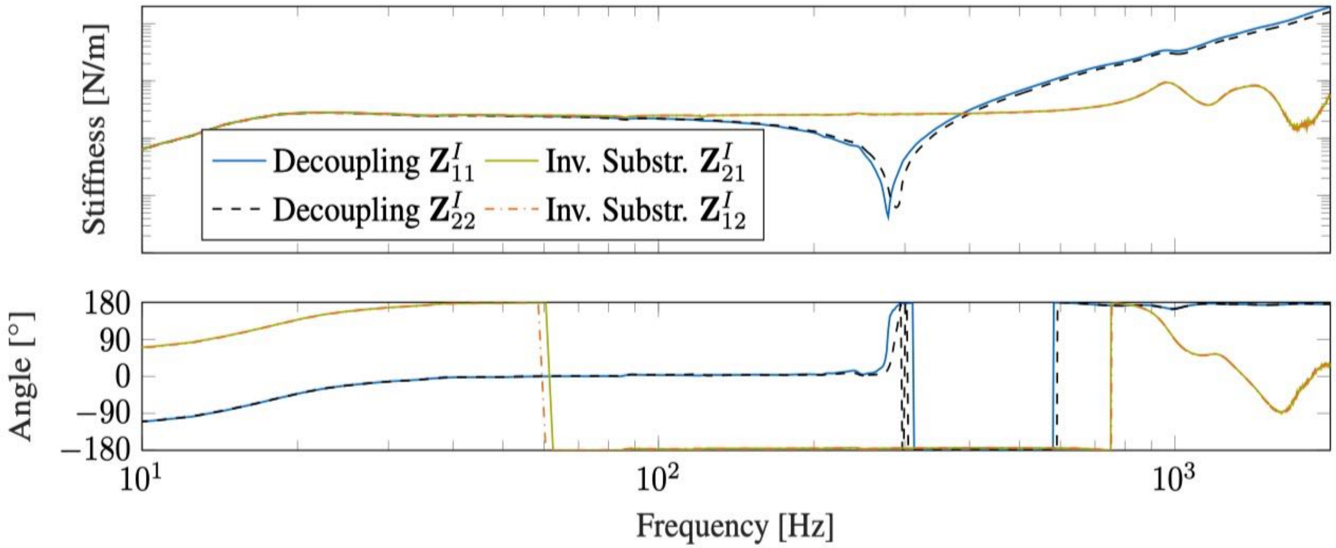
$$\left(\mathbf{Y}_{22}^{AJB}\right)^{-1} = \begin{bmatrix} \boldsymbol{Z}_{22}^{A} + \boldsymbol{Z}^{J} & -\boldsymbol{Z}^{J} \\ -\boldsymbol{Z}^{J} & \boldsymbol{Z}_{22}^{B} + \boldsymbol{Z}^{J} \end{bmatrix}$$



Axial direction: Comparison of hydropulse with inverse substructuring results.

Example: Rubber joint





Dual Decoupling

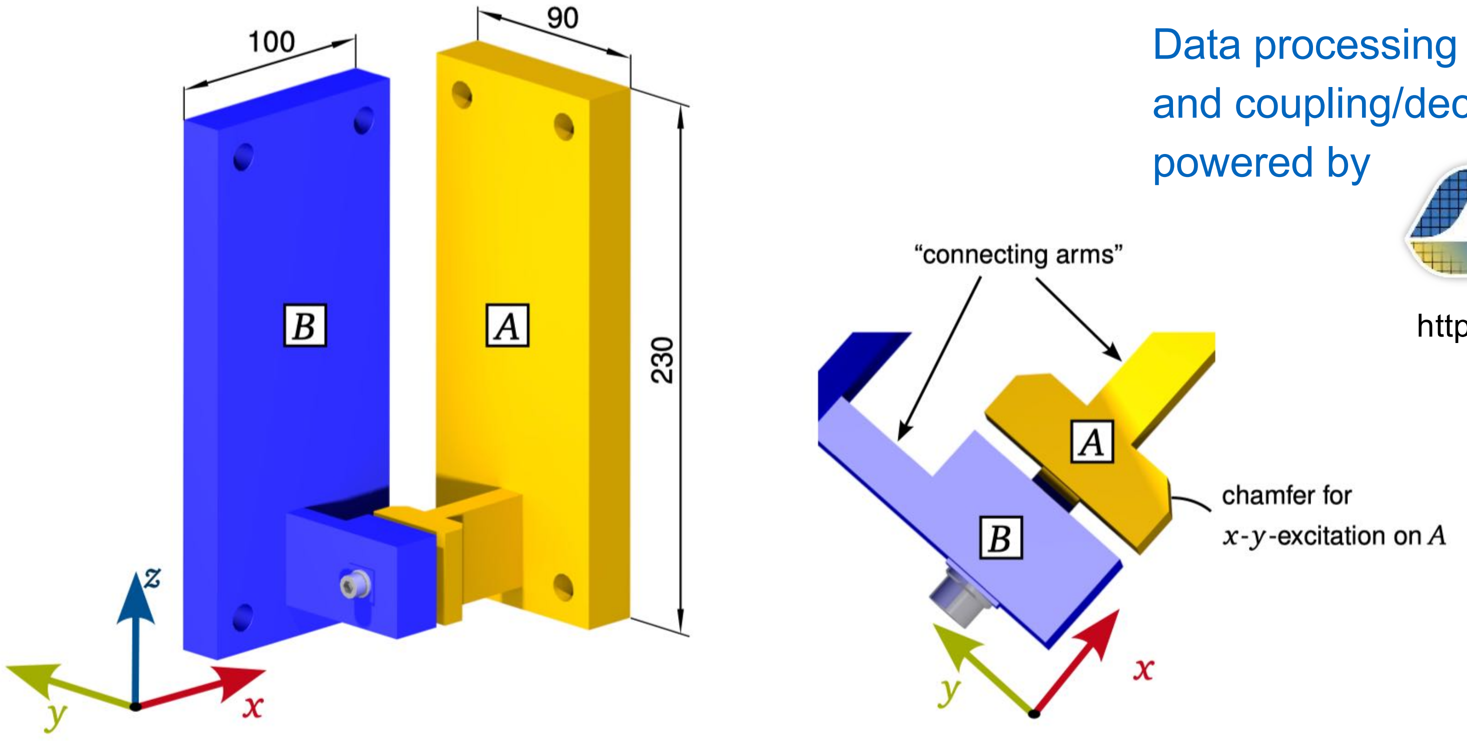
$$\boldsymbol{Y}^J_{\text{decoupled}} = \left(\boldsymbol{I} - \boldsymbol{Y}\boldsymbol{B}^T\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T\right)^{-1}\boldsymbol{B}\right)\boldsymbol{Y}$$

$$\boldsymbol{Y} = \begin{pmatrix} \boldsymbol{Y}^{AJB}_{qm} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{Y}^{A}_{qm} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{Y}^{B}_{qm} \end{pmatrix}$$

Example: Bolted joint [1]

Data processing
and coupling/decoupling
powered by pyFBS

https://gitlab.com/pyFBS/pyFBS

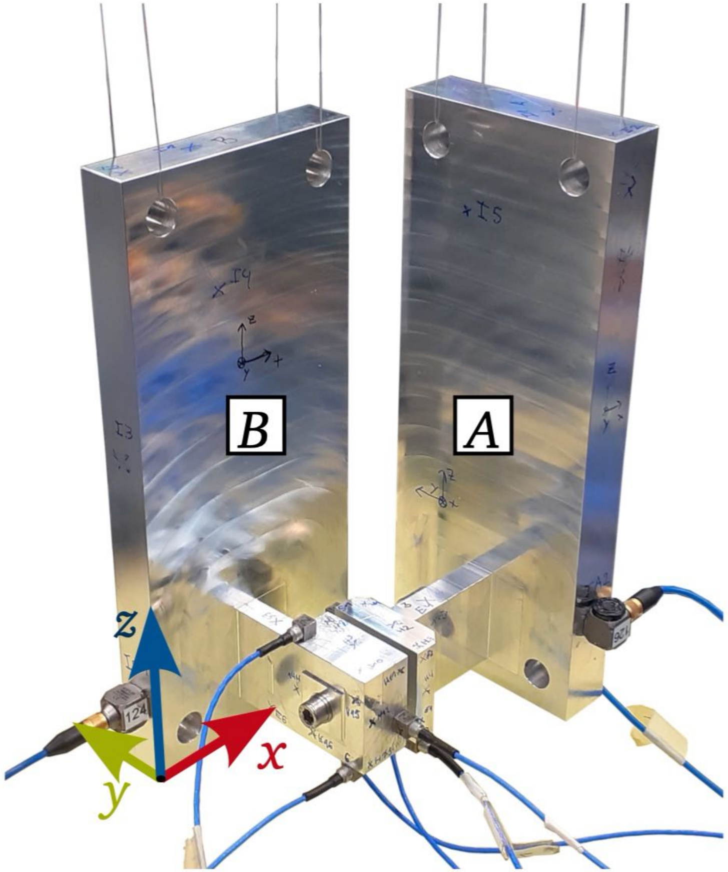


(a) Whole system.

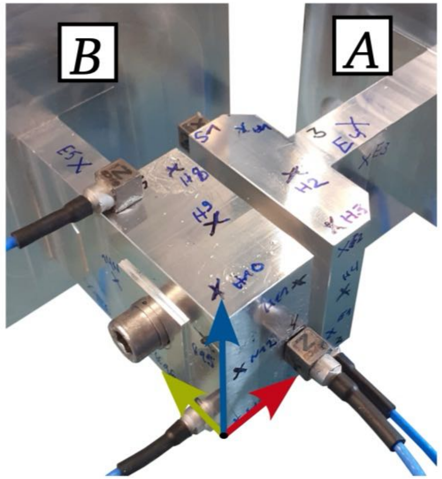
(b) Close-up of the contact.

[1] M. Kreutz, F. Trainotti, V. Gimpl, and D. J. Rixen. On the robust experimental multi-degree-of-freedom identification of bolted joints using frequency-based substructuring. *Mechanical Systems and Signal Processing*, 203:110626, 2023.

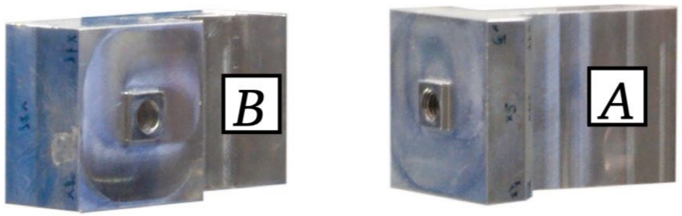
Example: Bolted joint [1]



(a) Measurement setup with suspension.



(b) Close-up of the assembled interface.

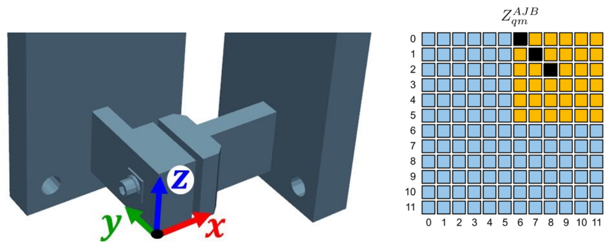


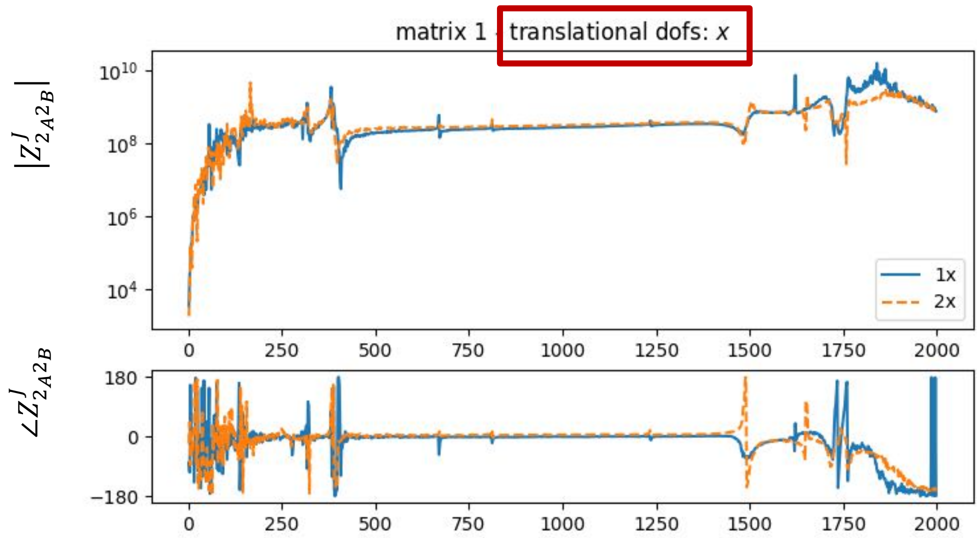
(c) Close-up on disassembled A and B.

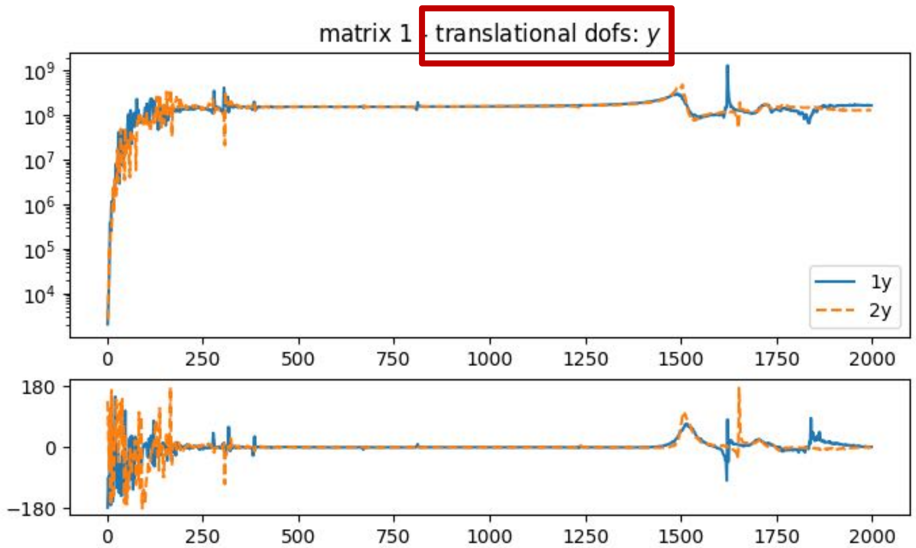
Example: Bolted joint [1]

Primal Inv.Substr.

$$\left(\mathbf{Y}_{22}^{AJB}\right)^{-1} = \begin{bmatrix} \boldsymbol{Z}_{22}^{A} + \boldsymbol{Z}^{J} & -\boldsymbol{Z}^{J} \\ -\boldsymbol{Z}^{J} & \boldsymbol{Z}_{22}^{B} + \boldsymbol{Z}^{J} \end{bmatrix}$$



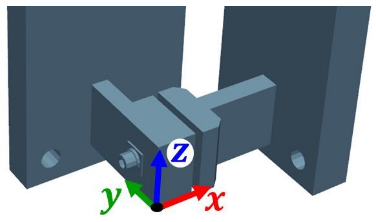


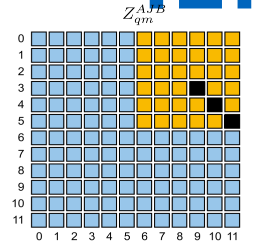


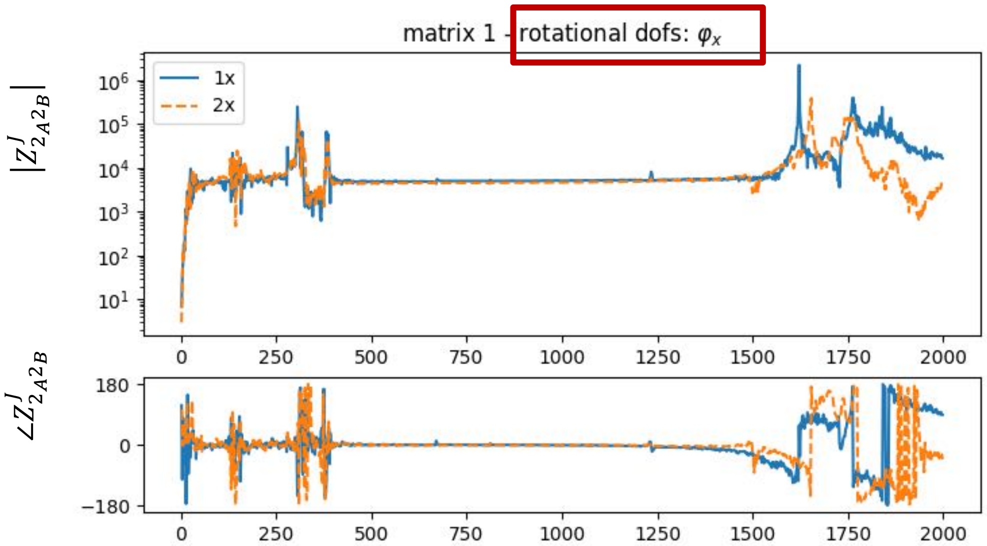
Example: Bolted joint [1]

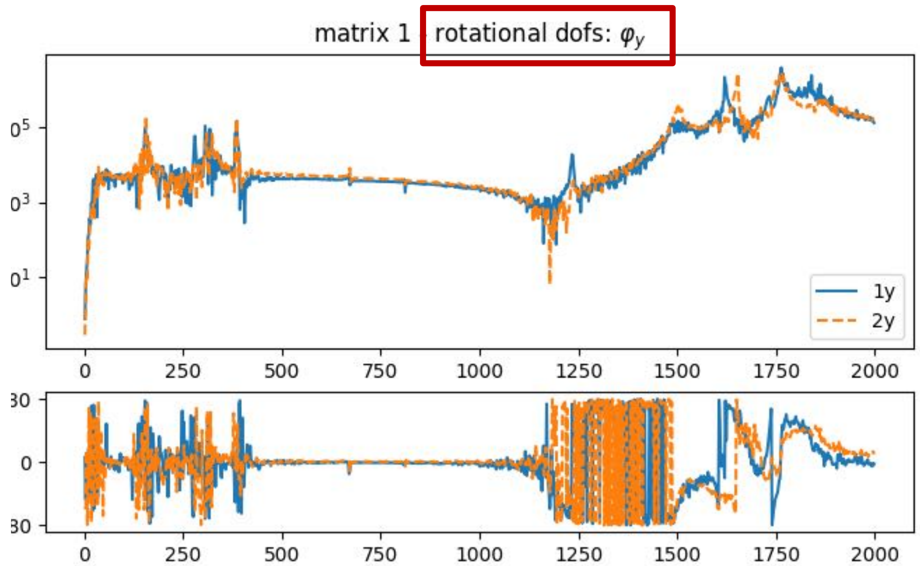
Primal Inv.Substr.

$$\left(\boldsymbol{Y}_{22}^{AJB}\right)^{-1} = \begin{bmatrix} \boldsymbol{Z}_{22}^{A} + \boldsymbol{Z}^{J} & -\boldsymbol{Z}^{J} \\ -\boldsymbol{Z}^{J} & \boldsymbol{Z}_{22}^{B} + \boldsymbol{Z}^{J} \end{bmatrix}$$



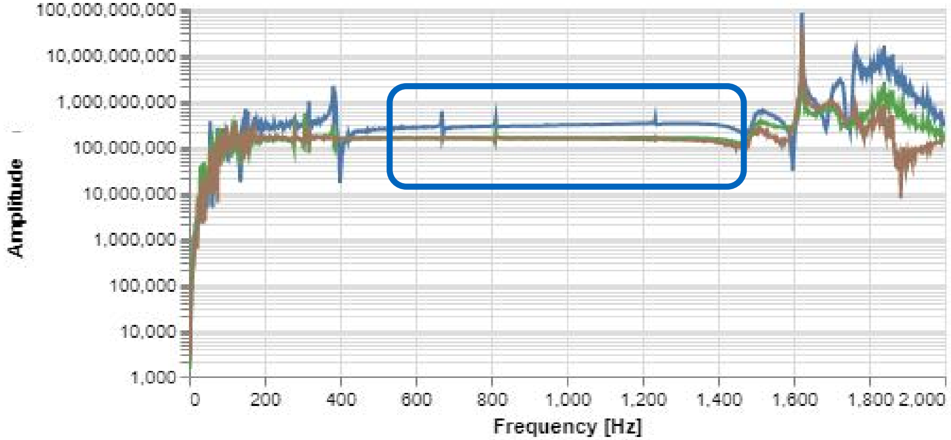






Identification

select *clean*
frequency range



fit parameters
for each direction

$$Z_{ii}(\Omega) = k_i + j\Omega d_i$$

Identified joint stiffness for two measurement sets. The system was disassembled and reassembled between the measurements.

	Stiffness first measurement set	Stiffness second measurement set	Change in stiffness
k_x	2.50×10^8 N m^{-1}	3.08×10^8 N m^{-1}	+23%
k_y	1.54×10^8 N m^{-1}	1.55×10^8 N m^{-1}	+1%
k_z	1.61×10^8 N m^{-1}	1.59×10^8 N m^{-1}	−1%
k_{φ_x}	5.10×10^3 N m rad^{-1}	4.66×10^3 N m rad^{-1}	−8%
k_{φ_y}	4.02×10^3 N m rad^{-1}	4.97×10^3 N m rad^{-1}	+24%
k_{φ_z}	6.30×10^3 N m rad^{-1}	5.67×10^3 N m rad^{-1}	−10%

And now some commercials ...

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